2020

PHYSICS (Honours)

Paper : PHYH - DC-5T

[CBCS]

Full Marks : 25 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

1. Answer any *five* questions : 2×5=10

(a) Locate and classify the singular points of the following equation.

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$$

(b) $I=\int_{-1}^{+1}x^3\,P_2(x)dx$ Evaluate I without actually performing the integration.

(c) Show that $P_n\left(-x\right)=\left(-1\right)^n P_n\left(x\right)$.

(d) Prove that $\Gamma\left(½\right)=\sqrt{\pi}$

(e) Express Bessel function $J_n\left(x\right)$ when n=0.

(f) For Beta function show that $B(n, m) = B(m,n)$.

(g) In the following Lagrangian $L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\right)-V(r)$, mention the cyclic coordinate. Find out the conjugate momentum to the cyclic co-ordinate.

2. Answer any *three* questions : 5×3=15

(a) Expand $f(x) = -x^2 (-\pi \le x \le \pi)$ in a Fourier series when the period is 2π and hence show that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$. 5

(b) (i) Write down the Rodrigues formula for Legendre Polynomial

(ii) $(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$

where $P_n(x)$ is Legendre's polynomials. 2+3

(c) Prove the orthogonality condition of Legendre's polynomials. 5

(d) Write down Laplace's equation $\nabla^2 \varphi = 0$ in spherical polar coordinates and separate the equation by the method of separation of variables. 5

(e) What is Gamma function ? Derive its relationship with Beta function. 5

(f) Derive the canonical equations of Hamiltonian. 5